Greek letters

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Refresher on the Black & Scholes' model

- Notations
  - C = call premium
  - P = put premium
  - S = spot price of the underlying asset
  - E = option's strike price
  - τ = time to expiration of the option
  - σ = volatility of the underlying asset
  - r = risk free rate
- Black & Scholes formula: C= $S.\Phi(d_1) - Ee^{-r\tau}\Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{S}{E}\right) + \left(r + \frac{\sigma^2}{2}\right).\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}\,, \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \int_{-\infty}^{x} f(t)dt$$

- Call-put parity: $P = C - S + Ee^{-r\tau}$

Greek letters' formulas

- Delta
  - Sensitity of the option's premium to a slight change in the spot price of the underlying asset
  - Delta of a call = $\frac{\delta C}{\delta S} = \Phi(d_1)$
  - Delta of a put = $\frac{\delta P}{\delta S} = \Phi(d_1)\text{-}1 = -\Phi(-d_1)$
- Vega
  - Sensitity of the option's premium to a slight change in the volatility of the underlying asset
  - Vega of a call = $\frac{\delta C}{\delta \sigma}$ = Vega of a put = $\frac{\delta P}{\delta \sigma} = S.f(d_1)\sqrt{\tau}$
- Rho
  - Sensitity of the option's premium to a slight change in the risk-free rate
  - Rho of a call = $\frac{\delta C}{\delta r} = \tau E e^{-r\tau}\Phi(d_2)$
  - Rho of a put = $\frac{\delta P}{\delta r} = -\tau E e^{-r\tau}\Phi(-d_2)$
- Theta
  - Sensitity of the option's premium to a slight change in the option's time to expiration
  - Theta of a call = $\frac{\delta C}{\delta \tau} = rEe^{-r\tau}\Phi(d_2) + Sf(d_1)\frac{\sigma}{2\sqrt{\tau}}$
  - Theta of a put = $\frac{\delta P}{\delta \tau} = -rEe^{-r\tau}\Phi(-d_2) + Sf(d_1)\frac{\sigma}{2\sqrt{\tau}}$

Numerical example

Spot price of the underlying asset : S	120	121	120	120	120
Strike price : E	100	100	100	100	100
Valuation date : t'	01/01/2019	01/01/2019	01/01/2019	02/01/2019	01/01/2019
Expiration date : t''	01/11/2019	01/11/2019	01/11/2019	01/11/2019	01/11/2019
Volatility : σ	20%	20%	21%	20%	20%
Risk free rate in discrete time : r'	6,00%	6,00%	6,00%	6,00%	7,07%
Time to expiration (in years) : t =(t''-t')/365	0,833	0,83	0,83	0,83	0,83
Risk free rate in continuous time : r = *ln* (1+r')	5,83%	5,83%	5,83%	5,83%	6,83%
$d_1 = \frac{ln\left(\frac{S}{E}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$	1,36	1,40	1,30	1,36	1,40
$d_2 = d_1 - \sigma\sqrt{\tau}$	1,17	1,22	1,11	1,17	1,22
Φ(d1)	0,9125	0,9195	0,9033	0,9126	0,9195
Φ(d2)	0,8797	0,8886	0,8662	0,8800	0,8886
Call premium: C = S.Φ(d_1)-E.exp(-rt).Φ(d_2)	**25,69**	**26,61**	**25,87**	**25,67**	**26,39**
Put premium: P = C-S+E.exp(-rt)	**0,95**	**0,87**	**1,14**	**0,95**	**0,86**
Gap on call premium		**0,92**	**0,18**	**-0,02**	**0,70**
Gap on put premium		-0,08	0,18	0,00	-0,09
$f(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$	0,16	0,15	0,17	0,16	0,15

Greek letters				
Delta				
Call				
Δ = Φ(d1)	**0,91**	0,92		
Change in delta		0,01		
Put				
Δ = -Φ(-d1)	-0,09	-0,08		
Change in delta		0,01		
Vega: call and put				
V for 100% = $f(d_1).S.\sqrt{\tau}$	17,42			
Véga for 1% = V / 100	0,17			
Theta		**Rhô**		
Call		**Call**		
T for 1 year	6,97	ρ for 100%		69,80
Théta for 1 day = / 365	0,02	Rhô for 1% = ρ /100		0,70
Put		**Put**		
T for 1 year	1,42	ρ for 100%		-9,54
Théta for 1 day = / 365	0,00	Rhô for 1% = ρ /100		-0,10

Α	α	alpha	Ι	ι	iota	Ρ	ρ	rho
Β	β	beta	Κ	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	Τ	τ	tau
Ε	ε	epsilon	Μ	μ	mu	Υ	υ	upsilon
Δ	δ	delta	Ν	ν	nu	Φ	φ	phi
Ζ	ζ	zeta	Ξ	ξ	xi	Χ	χ	chi
Η	η	eta	Ο	ο	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega