Credit Default Swaps

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1. Principles

- Insurance against a risk of default by a reference entity that also enables to speculate on the creditworthiness of reference entity
- Default = credit event
  - Bankruptcy / Failure to pay / Repudiation / Obligation default
  - Restructuring of debt
  - Moratorium
- Right granted to the buyer of a CDS to sell bonds issued by the reference entity for the principal value of the bond
  - Choice by the buyer of the cheapest to deliver bond, with the same priority
  - Possibility of cash settlement : payment by the seller of the CDS to the buyer
    - (100-Z)% of the nominal value where Z = middle of the reference bond's price, at an agreed date, just after the credit event
    - Or L.(1-RR) where L is the principal and RR the recovery rate
- Differences with an insurance contract
  - No required ownership of the underlying bonds: possible naked CDS except for Sovereign bonds
  - No need for the CDS' seller to be a regulated entity
  - No need for the CDS' seller to maintain reserves to cover the production sold
  - No need for the CDS' seller to disclose all known risks
  - Insurance companies' risk managed by loss reserves booking based on the law of large numbers and actuarial analysis versus hedging with other CDS and underlying bond markets
  - Markt to market of CDS, introducing income statement volatility which is not the case of insurance contracts
  - Assumed cancellation of an insurance contract when the buyer stops paying premium versus unwound of CDS contract
- Regular payments (quarterly payments in general) by the buyer to the seller
  - Until the end of the life of the CDS or till a credit event
  - Usual final accrual payment given payments in arrears: half of the annual payment due if default occurs halfway
- Watchdog in charge of CDS regulation : International Swaps and Derivative Association (ISDA)

2. Example of speculation

- Purchase of CDS by a hedge fund from a bank
  - Notional amount: 10 M€
  - CDS spread: 500 bps
  - Quarterly installments
  - Amount to be paid: 5% x 10 M€ = 500,000 € / year ie 125,000 € / quarter
  - Total payments assuming no default: 5 x 500,000 = 2.5 M€
- Action after 1 year assuming a widening of the spread from 500 bps to 1,500 bps
  - Sale of CDS for 1 year to the bank
  - Net Cash Flow for the hedge fund over the 2 first years:

$$(1x15\%x10\ M€\ -2\ x\ 5\%\ x\ 10\ M€\ ) = 1.5\ M€\ -1.0\ M€ = 0.5\ M€$$

3. Default intensity

3.1. Based on historical data (real world)

- Cumulative average default rates (%): 1970-1983

	1	2	3	4	5	7	10
Aaa	0.00	0.00	0.00	0.04	0.12	0.29	0.62
Aa	0.02	0.03	0.06	0.15	0.24	0.43	0.68
A	0.02	0.09	0.23	0.38	0.54	0.91	1.59
Baa	0.20	0.57	1.03	1.62	2.16	3.24	5.10
Ba	1.26	3.48	6.00	8.59	11.17	15.44	21.01
B	6.21	13.76	20.65	26.66	31.99	40.79	50.02
Caa	23.65	37.20	48.02	55.56	60.83	69.36	77.91

- Example for a Caa bond
  - Proba to default during Y3: 48.02%-37.20%=10.82% = P(A)
    - Non conditional probability of default
    - Proba of default seen from t=0
  - Survival proba till end of Y2: 100%-37.20%=62.80% = P(B)
  - Proba to default in Y3 assuming no default before:
    - $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{10.82\%}{62.80\%} = 17.23\%$
    - $P(A/B)$= default intensity for the 3rd year ie between the end of the 2nd year and the end of the 3rd year

- General case
  - $\bar{\lambda}(t)$: average default intensity between 0 et t
  - $\lambda(t)$: default intensity on time t
    - Definition of $\lambda(t)$ so that $\lambda(t).\Delta t$ = probability to default between t and $t + \Delta t$
  - $V(t)$: cumulative survival probability till time t ie probability not to have any default before time t
  - $V(t + \Delta t) - V(t) = -\lambda(t).V(t).\Delta t$
  - Going to the limit: $\frac{dV(t)}{V(t)} = -\lambda(t)\mathrm{dt}$
  - $\int_0^t \frac{dV(\tau)}{V(\tau)} = \int_0^t -\lambda(\tau)\mathrm{d}\tau = \ln[V(t)]$
  - $V(t) = e^{-\int_0^t \lambda(\tau)\mathrm{d}\tau} = e^{-t.\frac{1}{t-0}\int_0^t \lambda(\tau)\mathrm{d}\tau} = e^{-t.\bar{\lambda}(t)}$
  - $Q(t)$: proba of default till time t = real world probability
    - $Q(t) = 1 - V(t)$
    - $Q(t) = 1 - e^{-t.\bar{\lambda}(t)}$
  - $\bar{\lambda}(t) = \frac{1}{t}\ln[1 - Q(t)]$
- Example: calculation of $\bar{\lambda}(7)$ for a A bond:
  - $\bar{\lambda}(7) = \frac{1}{7}\ln[1 - Q(7)] = \frac{1}{7}\ln[1 - 0.0091] = 0.13\%$

3. Default intensity

3.2. Based on bond prices (risk neutral approach)

Rating	Real-world default probability per yr (bps)	Risk-neutral default probability per yr (bps)	Ratio	Difference
Aaa	4	67	16.8	63
Aa	6	78	13.0	72
A	13	128	9.8	115
Baa	47	238	5.1	191
Ba	240	507	2.1	267
B	749	902	1.2	153
Caa	1690	2130	1.3	440

- Notations
  - *s*: CDS' spread
  - *RR*: recovery rate
  - *h*: probability of default for a given year = default intensity
- Formula: $h = \frac{s}{1-RR}$
- Example:
  - RR = 40%
  - Risk free rate
    - 7Y swap rate – 10bp
    - 5.605% - 0.10% = 5.505%
  - Rate for 7Y A bonds: 6.274%
  - $h = \frac{6.274\% - 5.505\%}{1-40\%} = 1.28\%$
- Default intensity to be used
  - Valuation of CDS and sensitivity analysis of their price to changes in the credit risk: risk neutral probability
  - Estimate of potential losses in various scenarii: real world probability

4. Recovery rates

- Bond's recovery rate (RR) for a bond: price of the bond immediately after default as a percent of its face value
- Average RR by asset class according to Moody's (1982 -2003)
  - Senior secured 51.6%
  - Senior unsecured 36.1%
  - Senior subordinated 32.5%
  - Subordinated 31.1%
  - Junior subordinated 24.5%

5. Valuation example

Risk free rate (LIBOR)	5%
Recovery Rate given default	40%
Default occurrence	Halfway through a year
CDS payments	End of the year
Probability of default during a year conditional on no ea	2%
CDS spread = yearly payment assuming no default	s

Time (Years)	Default probability	Survival probability
1	0,0200	0,9800
2	0,0196	0,9604
3	0,0192	0,9412
4	0,0188	0,9224
5	0,0184	0,9039

PV of expected payments by the CDS' holder

Time (Years)	Survival probability	Expected Payment		Discount factor	PV of E(payment)	
1	0,9800	0,9800	s	0,9512	0,9322	s
2	0,9604	0,9604	s	0,9048	0,8690	s
3	0,9412	0,9412	s	0,8607	0,8101	s
4	0,9224	0,9224	s	0,8187	0,7552	s
5	0,9039	0,9039	s	0,7788	0,7040	s
Total					4,0704	s

PV of expected payoffs to the CDS' holder given default

Time (Years)	Default probability	Recovery rate = RR	Expected payoff	Discount factor	PV of E(payoff)
0,5	0,0200	40%	0,0120	0,9753	0,0117
1,5	0,0196	40%	0,0118	0,9277	0,0109
2,5	0,0192	40%	0,0115	0,8825	0,0102
3,5	0,0188	40%	0,0113	0,8395	0,0095
4,5	0,0184	40%	0,0111	0,7985	0,0088
Total					0,0511

PV of accrued payments assuming default

Time (Years)	Default probability	Expected payoff		Discount factor	PV of E(payoff)	
0,5	0,0200	0,0100	s	0,9753	0,0098	s
1,5	0,0196	0,0098	s	0,9277	0,0091	s
2,5	0,0192	0,0096	s	0,8825	0,0085	s
3,5	0,0188	0,0094	s	0,8395	0,0079	s
4,5	0,0184	0,0092	s	0,7985	0,0074	s
Total					0,0426	s

Sum of PV of payments by the CDS' holder	4,1130	s		
Sum of PV of payoffs to the CDS' holder	0,0511			
s	0,0124	=	124	bp

6. Valuation

- Notations
  - N: CDS' nominal value
  - t_i: quarter Nr i
  - s: spread
  - δ_i: discount factor for the payment at the end of the i^{th} quarter
  - p_i: survival probability over the $[t_{i-1}, t_i]$ period
  - RR: recovery rate

Description	PV (premium payment)	PV (Default payment)	Probability
Default at time t_1	0	$N(1-RR)\delta_1$	$1\text{-}p_1$
Default at time t_2	$-Ns\delta_1/4$	$N(1-RR)\delta_2$	$p_1(1-p_2)$
Default at time t_3	$-Ns(\delta_1+\delta_2)/4$	$N(1-RR)\delta_3$	$p_1p_2(1-p_3)$
Default at time t_4	$-Ns(\delta_1+\delta_2+\delta_3)/4$	$N(1-RR)\delta_4$	$p_1p_2p_3(1-p_4)$
No default	$-Ns(\delta_1+\delta_2+\delta_3+\delta_4)/4$	0	$p_1p_2p_3p_4$

$$CDS\ price = N(1-RR)[(1-p_1)\delta_1 + p_1(1-p_2)\delta_2 + p_1p_2(1-p_3)\delta_3 + p_1p_2p_3(1-p_4)\delta_4]$$
$$-Ns/4[p_1(1-p_2)\,\delta_1 + p_1p_2(1-p_3)(\delta_1+\delta_2) + p_1p_2p_3(1-p_4)(\delta_1+\delta_2+\delta_3) + p_1p_2p_3p_4(\delta_1+\delta_2+\delta_3+\delta_4)]$$

$$CDS\ price = N(1-RR)[(1-p_1)\delta_1 + p_1(1-p_2)\delta_2 + p_1p_2(1-p_3)\delta_3 + p_1p_2p_3(1-p_4)\delta_4]$$
$$-Ns/4[p_1\delta_1 + p_1p_2\delta_2 + p_1p_2p_3\delta_3 + p_1p_2p_3p_4\delta_4]$$

7. Forward CDS and options on CDS

- Forwards
  - Commitment to buy or sell a CDS at a future date on a given reference entity
  - In case of credit event, cancellation of the commitment
- Options
  - Payoff on expiration date based on the credit spread of the reference entity
  - Example: CDS call with a 280 bps strike and a 1Y maturity
    - If the reference entity’s spread in 1Y > 280 bps: exercise of the call
    - If the reference entity’s spread in 1Y < 280 bps: abandonment of the call
  - In case of credit event, cancellation of the commitment