CEMS Program

Greek Letters

Sensitivity analysis of an option premium

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Greek letters

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The partial derivatives of the Black and Scholes formula provide sensitivity indicators of the option's premium to small changes in some parameters: spot price of the underlying asset *S*, volatility of the underlying asset σ, time to expiration τ and risk-free rate *r*.
These parameters are often noted by Greek letters:

- delta (Δ) for the sensitivity to *S*
- vega (which is not a Greek letter) for the sensitivity to σ
- theta (θ) for the sensitivity to τ
- rho (ρ) for the sensitivity to *r*

As the Δ enables to build hedging strategies, a sensitivity of this Δ to a change in *S* is measured by the Gamma (Γ)

1. Delta

Let $\Delta = \frac{\partial C(S)}{\partial S}$
with:
∂C = change in the call premium
∂S = change in the spot price of the underlying asset.

Based on the Black and Scholes formula:

$$C = S\Phi(d_1) - Ee^{-r\tau}\Phi(d_2) \text{ with } d_1 = \frac{\ln\frac{S}{E}+(r+\frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \text{ and } d_2 = d_1 - \sigma\sqrt{\tau}$$

$\frac{\partial f(x)}{\partial x} = f'(x)$ hence, here:

$\frac{\partial C(S)}{\partial S} = C'(S)$ = partial derivative of C, S being the variable

$$\text{Moreover : } d_1 = \frac{\ln\frac{S}{E}+r\tau+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}} = \frac{\ln\frac{S}{E}+\ln e^{r\tau}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}} = \frac{\ln\frac{S}{Ee^{r\tau}}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}} = \frac{\ln\frac{S}{(Ee^{-r\tau})}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}$$

$$\text{Then: } \quad d_1 = \frac{\ln S - \ln Ee^{-r\tau}+\frac{\sigma^2}{2}}{\sigma\sqrt{\tau}}$$

$$\Delta = C'(S) = [S\Phi(d_1) - Ee^{-r\tau}\Phi(d_2)]' = 1.\Phi[d_1(S)] + S\{\Phi[d_1(S)]\}' - Ee^{-r\tau}\{\Phi[d_2(S)]\}'$$

as: $\{u[v(x)]\}' = u'[v(x)].\ v'(x)$.

Furthermore, if $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$ then $\Phi'(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} = f(x)$

Hence:

$\Delta = \Phi(d_1) + S.f[d_1(S)].d'_1(S) - Ee^{-r\tau}.f[d_2(S)].d'_2(S)$.

But: $d_2(S) = d_1(S) - \sigma\sqrt{\tau}$ then, $d'_2(S) = d'_1(S)$

And:

$$f(d_2) = f(d_1 - \sigma\sqrt{\tau}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(d_1-\sigma\sqrt{t})^2}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2} + d_1\sigma\sqrt{\tau} - \frac{\sigma^2\tau}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}.e^{d_1\sigma\sqrt{\tau}}.e^{-\frac{\sigma^2\tau}{2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}.e^{\ln\frac{S}{Ee^{-r\tau}} + \frac{\sigma^2\tau}{2}}.e^{-\frac{\sigma^2\tau}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}.\frac{S}{E.e^{-r\tau}}.e^{\frac{\sigma^2\tau}{2}}.e^{-\frac{\sigma^2\tau}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}.\frac{S}{E.e^{-r\tau}}$$

Hence:

$$\boxed{f(d_2) = f(d_1).\frac{S}{E.e^{-r\tau}}}$$

Then:

$\Delta = \Phi(d_1) + S.f(d_1).d'_1 - Ee^{-r\tau}.\ f(d_1).\frac{S}{E.e^{-r\tau}}.d'_1$.

Simplifying by $Ee^{-r\tau}$ and by $S.f(d_1).d'_1$:

$$\boxed{\text{Delta of the call} = \frac{\delta C}{\delta S} = \Phi(d_1)}$$

Considering 3 particular cases:

<u>1st case: call deeply *in the money*</u>

In other words, *S* is significantly higher than the present value of the strike price, based on continuous discounting.

Then: $S >> Ee^{-r\tau}$ or $\frac{S}{Ee^{-r\tau}} \to +\infty$.

$\Delta = \Phi(d_1) = \Phi\left[\frac{\ln\frac{S}{Ee^{-r\tau}} + \frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}\right] = \Phi(+\infty)$ as $\lim \ln x = +\infty$ when *x* is narrowing $+\infty$.

But : $\Phi(x) = \frac{1}{\sqrt{2\pi}}.\int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$; then: $\Phi(+\infty) = \frac{1}{\sqrt{2\pi}}.\int_{-\infty}^{+\infty} e^{-\frac{t^2}{2}} dt = 1$

Conclusion: the delta of a call that is deeply in the money is equal to 1.
The changes in the call and in the spot price of the underlying asset are alike. Then, the hedging of 1 share can be obtained by the sale of a deeply *in the money* call

2nd case: the call is deeply *out of the money*

In other words, S is significantly lower than the present value of the strike, based on a continuous discounting: S << Ee-rτ or: $\frac{S}{Ee^{-r\tau}} \to 0$

Then: $\Delta = \Phi(d_1) = \Phi[\frac{\ln\frac{S}{Ee^{-r\tau}}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}] = \Phi(-\infty)$

Finally: $\Delta = \frac{1}{\sqrt{2\pi}}.\int_{-\infty}^{-\infty} e^{-\frac{t^2}{2}}dt = 0$ or $\frac{\partial C(S)}{\partial S} = 0$

Conclusion

A portfolio of stocks can't be hedged with deeply *out of the money* calls

3rd case: the call is *at the money*

In other words: $S = Ee^{-r\tau}$

Then: $\Delta = \Phi(d_1) = \Phi[\frac{\ln\frac{S}{Ee^{-r\tau}}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}] = \Phi[\frac{\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}]$ as ln1 = 0

Hence: $\Delta = \Phi(d_1) = \Phi[\frac{\sigma\tau}{2\sqrt{\tau}}] = \Phi[\frac{\sigma\tau\sqrt{\tau}}{2\tau}] = \Phi[\frac{\sigma\sqrt{\tau}}{2}]$ the limit of which is $\Phi(0)$ when $\tau \to 0$.

Finally: $\Phi(0) = 0,5$ ie the limit of Δ is 0,5 when $\tau \to 0$.

Conclusion

If the change in the share price is 1 €, the change in the call premium is 0,5. Then, a portfolio made of 1 share can be hedged thanks to the sale of 2 calls.

Moreover, the call put parity enables to get the delta of a put. Indeed:

$P = C - S + E.e^{-r\tau}$.

Then:

Delta of the put $= \frac{\partial P}{\partial S} = \frac{\partial C}{\partial S} - \frac{\partial S}{\partial S} + E.\frac{\partial e^{-r\tau}}{\partial S}$ = Delta du call – 1 – 0 = $\Phi(d_1) - 1$

Finally:

Delta of the put = $-\Phi(-d_1)$

Example 1

The spot price of a stock is 120 €; this stock is the underlying asset of options, the strike price of which is 100€; the expiration date is 01/11/2019, the volatility is 20% and the T-Bonds rate is 6%.

C are P are respectively call and put premiums as at 01/01/2019. They can be determined by the Black and Scholes formula.

The risk free rate is: r = ln(1+0,06) = 0,0583 = 5,83%.
The time to expiration is $\tau = \frac{(01/11/2003 - 01/01/2003)}{365}$ = 0,833 year

$$d_1 = \frac{\ln\frac{120}{100} + (0{,}0583 + \frac{0{,}20^2}{2}).0{,}833}{0{,}20.\sqrt{0{,}833}} = 1{,}36$$

$d_2 = 1{,}36 – 0{,}20\sqrt{0{,}833} = 1{,}17$

C = 120.Φ(1,36)–100.$e^{-0{,}0583x0{,}833}$ Φ(1,17) = 25,69 €
P = 25,69 – 120 + 100. $e^{-0{,}0583x0{,}833}$ = 0,95 €.

The Delta of the call is Φ(1,36) = 0,91 and the delta of the put is -Φ(-1,17) = -0,09.

In other terms, a 1 € increase in the share price implies a 0,91 € increase in the call premium and a 0,09 € decrease in the put premium.

Indeed, the call and put premiums corresponding to a 121 € spot price of the underlying asset, based on the Black and Scholes formula, are respectively 26,61 € and 0,87 €:

$$d_1 = \frac{\ln\frac{121}{100} + (0{,}0583 + \frac{0{,}20^2}{2}).0{,}833}{0{,}20.\sqrt{0{,}833}} = 1{,}40 \text{ et } d_2 = 1{,}40 - 20\sqrt{0{,}833} = 1{,}22$$

C = 121.Φ(1,40)–100.$e^{-0{,}0583x0{,}833}$ Φ(1,22) = 26,61 €
P = 26,61 – 120 + 100. $e^{-0{,}0583x0{,}833}$ = 0,87 €.

The call premium has increased by 0,92 € (narrowing the 0,91 € delta), whereas the put premium has decreased by 0,08 € (narrowing the -0,09 € delta).

The slight c.0,01 € discrepancy between the changes in the premiums and the deltas relate to the way delta formulas have been obtained. Indeed, a delta is the derivative of the premium with respect to the spot price of the underlying asset. And the derivative has to be looked upon as the very small change in the premium implied by a very small change in the spot price of the underlying asset. Therefore, there would have been no discrepancy if the change in the share price had been significantly lower than 1€.

2. Gamma

A hedging strategy is based on the delta formula: $\Delta = \Phi(d_1)$.

But d_1 is a function of S; then Δ is always changing because of the changes in S.

Therefore, the hedging has to be adjusted accordingly with the change in S.

The Gamma measures the impact of the change in S on Δ.

In other words: $\Gamma = \frac{\partial \Delta}{\partial S} = \Delta'(S)$.

As $\Delta = \Phi(d_1)$, $\Delta' = \{\Phi[d_1(S)]\}' = f[d_1(S)].\ d_1'(S)$

As formerly, $f(x) = \frac{1}{\sqrt{2\pi}}.e^{\frac{-x^2}{2}}$ is the density of the standard normal distribution.

Then:

Γ of the call = $\frac{\partial \Delta}{\partial S} = \frac{f(d_1)}{S\sigma\sqrt{\tau}}$ with $d_1 = \frac{\ln\frac{S}{Ee^{-r\tau}} + \frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}$ and $f(x)$ $\frac{1}{\sqrt{2\pi}}.e^{\frac{-x^2}{2}}$

The Gamma measures the impact of the change in the spot price of the underlying asset on the number of calls to be sold to hedge a portfolio of shares.

Γ is maximum when:

- τ is narrowing 0;

- $f(d_1)$ is maximum ie $d_1 = 0$. This happens for low maturity options (τ is narrowing 0), and when $\frac{S}{Ee^{-r\tau}} = 1$ ie $S = Ee^{-r\tau}$ which means that the option is at the money.

Conclusion

The daily adjustments are the more important as hedging is based on short maturity calls and / or by *at the money* calls.

Thanks to the call put parity:

Gamma of the put = $\frac{\partial^2 P}{\partial S^2} = \frac{\partial^2 C}{\partial S^2} - \frac{\partial^2 S}{\partial S^2} + E.\ \frac{\partial^2 e^{-r\tau}}{\partial S^2}$
= Gamma of the call – 0 – 0
= Gamma of the call

Finally: Gamma du put = gamma du call = $\frac{f(d_1)}{S\sigma\sqrt{\tau}}$

Example 2

Coming back to example 1

d_1 = 1,36:

$$f(d_1) = \frac{1}{\sqrt{2\pi}}.e^{\frac{-1,36^2}{2}} = 0,16$$

Gamma of the call and of the put = $\frac{0,16}{120x0,20\sqrt{0,833}}$ = 0,01 €.

This amount is almost equal to the change in the deltas of both options: indeed, assuming a 1 € increase in the share price from 120 € to 121 €:

Delta of the call = Φ(1,40) = 0,92 versus 0,91 assuming a 120 € share price

Delta of the put = -Φ(-1,22) = -0,08 versus -0,09 assuming a 120 € share price

3. Vega

The vega measures the impact of the change in σ on the option premium:

Vega = $\frac{\partial C}{\partial \sigma}$ = C'(σ)

with:

C = $S\Phi(d_1)–Ee^{-r\tau}\Phi(d_2)$ with $d_1 = \frac{\ln\frac{S}{Ee^{-r\tau}}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}$ and = $d_1-\sigma\sqrt{\tau}$

C'(σ) = $S\{\Phi[d_1(\sigma)]\}'-Ee^{-r\tau}\{\Phi[d_2(\sigma)]\}'$

= $S.f[d_1(\sigma)].d'_1(\sigma) - Ee^{-r\tau}.f[d_2(\sigma)].d'_2(\sigma)$

But: $f(d_2) = f(d_1).\frac{S}{E.e^{-r\tau}}$

And : $d_2(\sigma)= d_1(\sigma)- \sigma\sqrt{\tau}$; hence: $d'_2(\sigma)= d'_1(\sigma) - \sqrt{\tau}$

Then:

C'(σ) = $S.f(d_1).d'_1 - Ee^{-r\tau}.\ f(d_1).\frac{S}{E.e^{-r\tau}}.(d'_1 - \sqrt{\tau})$.

= $S.f(d_1).d'_1 - Ee^{-r\tau}.\ f(d_1).\frac{S}{E.e^{-r\tau}}.\ d'_1 + Ee^{-r\tau}.\ f(d_1).\frac{S}{E.e^{-r\tau}}.\ \sqrt{\tau}$.

Simplifying by $Ee^{-r\tau}$ then by $S.f(d_1).d'_1$:

Vega of the call = $\frac{\delta C}{\delta \sigma}$ = $S.f(d_1).\sqrt{\tau}$

The Vega is maximum when

- $f(d_1)$ is maximum ie when $d_1 = 0$. Then the option is at the money;
- τ is very high which means that the expiration date is far from now.

Thanks to the call-put parity

Vega du put = $\frac{\partial P}{\partial \sigma} = \frac{\partial C}{\partial \sigma} - \frac{\partial S}{\partial \sigma} + E.\frac{\partial e^{-r\tau}}{\partial \sigma}$ = Véga du call – 0 – 0

Finally: **Vega of the call = vega of the put = $S.f(d_1).\sqrt{\tau}$**

Example 3

Coming back to example 1, the Vega measures the change in the options premium for a 1 ie 100% increase in volatility

$f(d_1) = 0{,}16$

Then:

Vega of the call and of the put = $120 x 0{,}16 x \sqrt{0{,}833}$ = 17 €.

Then, for a 1% increase in volatility, the vega is 0,17 €.

This amount is not far from the change in the options' premiums.

Indeed, assuming a 120 € spot price of the underlying asset and a 21% volatility:

$$d_1 = \frac{\ln\frac{120}{100} + (0{,}0583 + \frac{0{,}21^2}{2}).0{,}833}{0{,}21.\sqrt{0{,}833}} = 1{,}30$$

$d_2 = 1{,}30 - 0{,}21\sqrt{0{,}833} = 1{,}11$

$C = 120.\Phi(1{,}30) - 100.e^{-0{,}0583x0{,}833}\,\Phi(1{,}11) = 25{,}87$ €.

$P = 25{,}87 - 120 + 100.\ e^{-0{,}0583x0{,}833} = 1{,}14$ €.

The call premium is increased by 0,18 € from 25,69 € to 25,87 €, in line with the Vega

The put premium is increased by 0,18 € from 0,95 € to 1,14 €, whereas the Vega is 0,17 €.

4. Theta

The Theta measures the impact of the change in τ on the option premium:

Theta = $\theta = \frac{\partial C}{\partial \tau}$ = C'(τ)

with C = $S\Phi(d_1) - Ee^{-r\tau}\Phi(d_2)$

θ = C'(τ) = $S\{\Phi[d_1(\tau)]\}' + r.Ee^{-r\tau}\Phi[d_2(\tau)] - Ee^{-r\tau}\{\Phi[d_2(\tau)]\}'$

with $d_1 = \frac{\ln\frac{S}{Ee^{-r\tau}} + \frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}$ and $d_2(\tau) = d_1(\tau) - \sigma\sqrt{\tau}$ hence: : $d'_2(\tau) = d'_1(\tau) - \frac{\sigma}{2\sqrt{\tau}}$

Moreover: $\{\Phi[d_1(\tau)]\}' = f[d_1(\tau)].d'_1(\tau)$

And: $\{\Phi[d_2(\tau)]\}' = f[d_2(\tau)].d'_2(\tau)$

Then:

θ = C'(τ) = $S.f[d_1(\tau)].d'_1(\tau) + r.Ee^{-r\tau}\Phi[d_2(\tau)] - Ee^{-r\tau}.f[d_2(\tau)].d'_2(\tau)$

Furthermore: $f(d_2) = f(d_1).\frac{S}{E.e^{-r\tau}}$

In that case:

$$\theta = S.f[d_1(\tau)].d'_1(\tau) + r.Ee^{-r\tau}\Phi[d_2(\tau)] - Ee^{-r\tau}.\ f[d_1(\tau)].\frac{S}{E.e^{-r\tau}}.\left[d'_1(\tau) - \frac{\sigma}{2\sqrt{\tau}}\right]$$

$$= S.f[d_1(\tau)].d'_1(\tau) + r.Ee^{-r\tau}\Phi[d_2(\tau)] - S.f[d_1(\tau)].\left[d'_1(\tau) - \frac{\sigma}{2\sqrt{\tau}}\right].$$

Simplifying by $S.f[d_1(\tau)].d'_1(\tau)$:

Theta of the call $= \frac{\partial C}{\partial \tau} = r.Ee^{-r\tau}\Phi(d_2) + S.f(d_1).\frac{\sigma}{2\sqrt{\tau}}$

Thanks to the call put parity:

Theta of the put $= \frac{\partial P}{\partial \tau} = \frac{\partial C}{\partial \tau} - \frac{\partial S}{\partial \tau} + E.\frac{\partial e^{-r\tau}}{\partial \tau}$ = Theta of the call – 0 -r $E.e^{-r\tau}$

$$= r.Ee^{-r\tau}\Phi[d_2(\tau)] + S.f[d_1(\tau)].\frac{\sigma}{2\sqrt{\tau}} - rE.e^{-r\tau}$$

$$= r.Ee^{-r\tau}\{\Phi[d_2(\tau)] - 1\} + S.f[d_1(\tau)].\frac{\sigma}{2\sqrt{\tau}}$$

Theta of the put $= -r.Ee^{-r\tau}\Phi(-d_2) + S.f(d_1).\frac{\sigma}{2\sqrt{\tau}}$

Example 4

Coming back to example 1:

Theta of the call = 0,0583 x 100e$^{-0,0583 \text{ x } 0,833}$$\Phi(1{,}36)+100x0{,}16x\dfrac{0{,}20}{2\sqrt{0{,}833}}$ = 6,97 €

Theta of the put = -0,0583 x 100e$^{-0,0583 \text{ x } 0,833}$$\Phi(1{,}36)+100x0{,}16x\dfrac{0{,}20}{2\sqrt{0{,}833}}$ = 1,42 €

Theta of the call for 1 day: = $-\dfrac{6{,}97}{365}$ = -0,02 €

Theta of the put for 1 day = - $\dfrac{1{,}42}{365}$ = -0,00 €

Assuming the valuation is performed on 02/01/2019 (one day nearer the expiration date):

$$\tau = \frac{(01/11/2003 - 02/01/2003)}{365} = 0{,}830 \text{ year}$$

$$d_1 = \frac{\ln\frac{120}{100} + (0{,}0583 + \frac{0{,}20^2}{2}).0{,}830}{0{,}20.\sqrt{0{,}830}} = 1{,}36$$

d_2 = 1,36 –0,20 $\sqrt{0{,}830}$ = 1,17

C = 120.Φ(1,36)–100.e$^{-0,0583x0,830}$ Φ(1,17) = 25,67 €.

P = 25,69 – 120 + 100. e$^{-0,0583x0,830}$ = 0,95 €.

The call premium is reduced from 25,69 € to 25,67 €, ie by 0,02 €, corresponding to the theta of the call.

The put is almost unchanged, which is consistent with a c.0 € theta

5. Rhô

The theta measures the impact of the change in *r* on the option premium:

Rho = $\rho = \frac{\partial C}{\partial r}$ = C'(*r*)

with C = SΦ(d_1)–E$e^{-r\tau}$Φ(d_2).

ρ = C'(*r*) = S{Φ[d_1(r)]}'+ τ E$e^{-r\tau}$Φ[d_2(r)]-E$e^{-r\tau}${Φ[d_2(r)]}'

With $d_1 = \frac{\ln\frac{S}{Ee^{-r\tau}}+\frac{\sigma^2\tau}{2}}{\sigma\sqrt{\tau}}$ and $d_2(\tau) = d_1(\tau)-\sigma\sqrt{\tau}$ hence : d'$_2$(r) = d'$_1$®

Moreover: Φ[d_1(r)]}'= *f*[d_1(r)].d'$_1$(r)

And:{Φ[d_2(r)]}'= f[d_2(r)].d'$_2$(r)

Then:

ρ = C'(r) = S.f[d_1(r)].d'$_1$(r)+ τ .E$e^{-r\tau}$Φ[d_2(r)]-E$e^{-r\tau}$.*f*[d_2(r)].d'$_2$(r)

Furthermore:

$$f(d_2) = f(d_1).\frac{S}{E.e^{-r\tau}}$$

In that case:

$$\rho = S.f[d_1(r)].d'_1(r) + \tau .Ee^{-r\tau}\Phi[d_2(r)] - Ee^{-r\tau} .\ f[d_1(r)].\frac{S}{E.e^{-r\tau}} .\ d'_1(r)$$

Simplifying by $S.f[d_1(r)]$.d'$_1$(r) :

Rho of the call = $\frac{\partial C}{\partial r} = \tau$ .E$e^{-r\tau}$Φ(d_2)

Thanks to the call put parity:

Rho of the put = $\frac{\partial P}{\partial r} = \frac{\partial C}{\partial r} - \frac{\partial S}{\partial r} + E.\frac{\partial e^{-r\tau}}{\partial \tau} r$ = Rho of the call – 0 - τ . $E.e^{-r\tau}$

= τ .E$e^{-r\tau}$Φ(d_2) - τ . $E.e^{-r\tau}$

= - τ . $E.e^{-r\tau}$ [1-Φ(d_2)]

Finally:

Rho of the put = - τ . $E.e^{-r\tau}$.Φ(-d_2)

Exemple 5

Coming back to example 1:

Rho of the call = 0,833 x 100.$e^{-0,583x0,833}\Phi(1,17)$ = 70 €

Rho of the put = 0,833 x 100.$e^{-0,583x0,833}\Phi(-1,17)$ = 0,833 x 100.$e^{-0,583x0,833}[\Phi(1,17)-1]$
Rho of the put = 10 €

For a 1% change in the risk-free rate:

Rho of the call = $\frac{70}{100}$ = 0,70 €

Rho of the put = - $\frac{10}{100}$ = -0,10 €

Assuming the risk-free rate is increased to 6,83%:

$$d_1 = \frac{\ln\frac{120}{100} + (0{,}0683 + \frac{0{,}20^2}{2}).0{,}833}{0{,}20.\sqrt{0{,}833}} = 1{,}40$$

$d_2 = 1{,}40 - 0{,}20\sqrt{0{,}833} = 1{,}22$

C = 120.$\Phi(1,40)$–100.$e^{-0,0683x0,833}$ $\Phi(1,27)$ = 26,39 €

P = 26,39 – 120 + 100. $e^{-0,0683x0,833}$= 0,86 €.

The call premium is increased by 0,70 € from 25,69 € to 26,39 €, which corresponds to the rho of the call.

The put premium is reduced by 0,09 € from 0,95 € à 0,86 €, in line with the -0,10 € rho of the put.

Appendix: detailed calculations

Spot price of the underlying asset : S	120	121	120	120	120
Strike price : E	100	100	100	100	100
Valuation date : t'	01/01/2019	01/01/2019	01/01/2019	02/01/2019	01/01/2019
Expiration date : t''	01/11/2019	01/11/2019	01/11/2019	01/11/2019	01/11/2019
Volatility : σ	20%	20%	21%	20%	20%
Risk free rate in discrete time : r'	6,00%	6,00%	6,00%	6,00%	7,07%
Time to expiration (in years) : t =(t''-t')/365	0,833	0,83	0,83	0,83	0,83
Risk free rate in continuous time : r = *ln* (1+r')	5,83%	5,83%	5,83%	5,83%	6,83%
$d_1 = \dfrac{ln\left(\frac{S}{E}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$	1,36	1,40	1,30	1,36	1,40
$d_2 = d_1 - \sigma\sqrt{\tau}$	1,17	1,22	1,11	1,17	1,22
Φ(d1)	0,9125	0,9195	0,9033	0,9126	0,9195
Φ(d2)	0,8797	0,8886	0,8662	0,8800	0,8886
Call premium: C = S.Φ(d_1)-E.exp(-rt).Φ(d_2)	**25,69**	**26,61**	**25,87**	**25,67**	**26,39**
Put premium: P = C-S+E.exp(-rt)	**0,95**	**0,87**	**1,14**	**0,95**	**0,86**
Gap on call premium		**0,92**	**0,18**	**-0,02**	**0,70**
Gap on put premium		-0,08	0,18	0,00	-0,09
$f(d_1) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$	0,16	0,15	0,17	0,16	0,15
Greek letters					
Delta					
Call					
Δ = Φ(d1)	**0,91**	0,92			
Change in delta		0,01			
Put					
Δ = -Φ(-d1)	-0,09	-0,08			
Change in delta		0,01			
Vega: call and put					
V for 100% = $f(d_1).S.\sqrt{\tau}$	17,42				
Véga for 1% = V / 100	0,17				
Theta					
Call					
T for 1 year	6,97				
Théta for 1 day = / 365	0,02				
Put					
T for 1 year	1,42				
Théta for 1 day = / 365	0,00				
Rhô					
Call					
ρ for 100%	69,80				
Rhô for 1% = ρ /100	0,70				
Put					
ρ for 100%	-9,54				
Rhô for 1% = ρ /100	-0,10				
Gamma (call and put) = $\Gamma = \dfrac{f(d_1)}{S.\sigma.\sqrt{\tau}}$	0,01				