Risk and return

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There are several ways for an investor to measure the risk: assuming any incremental risk implies an additional return, the standalone risk can be measured based on the buy and hold return (BHR); it can also rely on the equity's or on the firm's assets' volatility and on the distance to default. Moreover, the systemic risk can be based on the SRISK and the marginal expected shortfall (MES) indicators. These various risk parameters are presented hereafter.

1. BHR

The buy and hold return (BHR) is commonly used in the finance literature, to measure the standalone investment risk inside an industry. It is, for example, considered by Moussu and Petit-Romec (2017) when they establish that ROE is a strong predictor of bank standalone risk during the 2007-2008 crisis.

The *BHR* formula is the following one:

$$1 + BHR = \prod_{t=1}^{n}(1 + r_t)$$

Where r_t is the continuous daily return and *n* is the number of returns that are considered in the sample of listed share prices.

2. Distance to default

The distance to default (*D2D*), considered by Moody's in its ratings' analyses has been proposed by Crosbie and Bohn (2003). They assume that the firm's assets, hereafter noted *V*, define a geometric Brownian motion: $dV = \mu V dt + \sigma_V V dz$ where μ corresponds to the drift, σ_V to the assets' volatility and dz to a random variable that obeys a standard normal distribution.

According to Black & Scholes (1973), the shareholders have a call on the firm's assets, the strike price being a zero-coupon bond, the amount of which is D, to be repaid on maturity in *t* years. The equity value, noted E, is the premium of this call. Then, based on Ito's lemma:

$$dE(V,t) = \left[\frac{\partial E}{\partial t} + \mu S \frac{\partial E}{\partial V} + \frac{1}{2}\frac{\partial^2 E}{\partial V^2}{\sigma_V}^2 V^2\right] dt + \frac{\partial E}{\partial V}\sigma_V V dz$$

Assuming $E(V,t) = \ln(V), \frac{\partial E}{\partial t} = 0, \frac{\partial E}{\partial V} = \frac{1}{V}, \frac{\partial^2 E}{\partial V^2} = -\frac{1}{V^2}$ and $d\ln V = \left(\mu - \frac{{\sigma_V}^2}{2}\right) dt + \sigma_V dz$

Then $\ln V_t \hookrightarrow \mathcal{N}\left[\ln V_0 + \left(\mu - \frac{{\sigma_V}^2}{2}\right) dt, {\sigma_V}^2 dt\right]$ and $V_t = V_0 e^{\left(\mu - \frac{{\sigma_V}^2}{2}\right)dt + \sigma_V dz_t}$

The assets' return, in discrete time is: $r = \frac{V_t}{V_0} - 1$; in continuous time, their return is $i = ln(1 + r) = ln\left(\frac{V_t}{V_0}\right)$. The firm defaults when $V_t = D \Leftrightarrow \frac{V_t}{V_0} = \frac{D}{V_0} \Leftrightarrow ln\left(\frac{V_t}{V_0}\right) = ln\left(\frac{D}{V_0}\right) \Leftrightarrow i = ln\left(\frac{D}{V_0}\right)$

As $V_t = V_0 e^{\left(\mu - \frac{{\sigma_V}^2}{2}\right)dt + \sigma_V dz_t}$, $\ln\left(\frac{V_t}{V_0}\right) = i = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma dz_t$

The expected return $E(i) = E\left[\left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dz_t\right] = \left(\mu - \frac{\sigma^2}{2}\right)dt$ as $E(dz_t) = 0$ *and* $dt = t - 0 = t$. And the default risk is the lower as the expected return is higher than the return driving to default. In other term the higher the difference $E(i) - i = \left(\mu - \frac{\sigma^2}{2}\right)t - \ln\left(\frac{D}{V_0}\right) = \ln\left(\frac{V_0}{D}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t$, the lower the risk of default

The *D2D* corresponds to this difference expressed in number of standard deviations by unit of time. Then: $D2D = \frac{\ln\left(\frac{V_0}{D}\right)+\left(\mu-\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} = d_2$ of the customary Black & Scholes formula.

This enables to get the probability *p* of bankruptcy that can be defined by $p = P(V_t < D)$

As $V_t = V_0 e^{\left(\mu-\frac{\sigma_V{}^2}{2}\right)dt+\sigma_V dz}$, $p = P\left(V_0 e^{\left(\mu-\frac{\sigma_V{}^2}{2}\right)dt+\sigma_V dz} < D\right) = P\left(e^{\left(\mu-\frac{\sigma_V{}^2}{2}\right)dt+\sigma_V dz} < \frac{D}{V_0}\right)$

$$p = P\left(\left(\mu - \frac{\sigma_V{}^2}{2}\right)dt + \sigma_V \varepsilon\sqrt{dt} < ln\left(\frac{D}{V_0}\right)\right)$$

$$p = P\left(\varepsilon < \frac{ln\left(\frac{D}{V_0}\right) - \left(\mu - \frac{\sigma_V{}^2}{2}\right)dt}{\sigma_V\sqrt{dt}}\right) = P\left(\varepsilon < -\frac{ln\left(\frac{V_0}{D}\right) + \left(\mu - \frac{\sigma_V{}^2}{2}\right)dt}{\sigma_V\sqrt{dt}}\right)$$

$$p = \Phi\left(-\frac{ln\left(\frac{V_0}{D}\right) + \left(\mu - \frac{\sigma_V{}^2}{2}\right)dt}{\sigma_V\sqrt{dt}}\right) = \Phi(-d_2) = \Phi(-D2D)$$

The *D2D* and the probability of bankruptcy relies on the knowledge of *V* and σ_V.

Assuming the equity value defines a geometric Brownian motion, $dE = \mu E dt + \sigma_E E dz$

Moreover, $dE(V,t) = \left[\frac{\partial E}{\partial t} + \mu V\frac{\partial E}{\partial V} + \frac{1}{2}\frac{\partial^2 E}{\partial V^2}\sigma_V{}^2V^2\right]dt + \frac{\partial E}{\partial V}\sigma_V V dz.$

Then, equalizing the random terms of the 2 above equations and simplifying by *dz*: $\sigma_E E = \frac{\partial E}{\partial V}\sigma_V V$

As $\frac{\partial E}{\partial V} = \Phi(d_1) = \Phi\left[\frac{ln\left(\frac{V}{D}\right)+\left(r+\frac{\sigma_A{}^2}{2}\right)\tau}{\sigma_A\sqrt{\tau}}\right]$, *V* and σ_V are the 2 unknowns of the following non linear system of 2 equations, the first one being the call's premium as developed by Merton (1974),

$$\begin{cases} E = V.\Phi\left[\frac{\ln\left(\frac{V}{D}\right) + \left(r + \frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right] - De^{-r\tau}\Phi\left[\frac{\ln\left(\frac{V}{D}\right) + \left(r - \frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right] \\ \sigma_E E = \Phi\left[\frac{ln\left(\frac{V}{D}\right) + \left(r + \frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right]\sigma_V V \end{cases}$$

This system is solved thanks to Excel's solver.

Chan-Lau and Sy (2006) have highlighted financial institutions in general, banks in particular have a relationship with default which is different from corporate bodies. Indeed, a bank's bankruptcy occurs after a regulator's actions have been implemented. Banks must respect a solvency ratio which consists in dividing its equity, taking some solvency restatements into account, by its risk weighted assets. Then, the direct application of D2D to a bank has 2 important limits: on the one hand, the risk that is implied by the leverage is different from a corporate entity's as the bank's business model consists in being an intermediary player between lenders and borrowers. Then a bank can have the same rating as a corporate entity whereas the bank's leverage is meaningfully higher. On the other hand, the D2D assumes that equity is a buffer, but regulators and supervisors are used to implementing measures before the assets' value falls below the debt amount. Then, the debt is not the right reference for bank's default. Therefore, Chan-Lau propose to turn the D2D into a distance to capital (D2C) that replaces D by λL where λ is a correction factor and L is the sum of the short-term debt and 50% of half of the long-term debt. Then:

$$D2C = \frac{\ln\left(\frac{V}{\lambda L}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

Alternatively, Liu, Papakirykos and Yuan (2004) propose a simpler formula of the D2C:

$$D2C = \frac{\left(\frac{V - \lambda L}{V}\right)}{\sigma\sqrt{t}}$$

In both formulae, $\lambda = \frac{1}{1-CAR}$, the CAR, or capital adequacy ratio, being the required solvency ratio. Assuming that the RWA are equal to 100% of the total assets, $CAR = \frac{E}{V}$

However, Harada, Ito and Takahashi (2010) who examine whether the D2D can be useful to anticipate the failure of a bank in a near future favour this measure over the D2C as the D2D is a basic and widely used measure of credit risk assessment in literature. Analysing the evolution of the D2D of 8 Japanese banks before their failure in the 90s and at the beginning of the 21st century, they propose a breakdown between banks' short term and long-term liabilities to take into account the specificity of banks' financial structure compared to corporate entities. For example, term deposits are included in the short-term debt as they can be withdrawn should depositors forego part of the accrued interest in the background of a bank run.

They propose to set the time horizon, τ, to 1 year which is a customary assumption when information of the liabilities' maturity structure is unavailable. This assumption will be included in the empirical study that is developed hereafter. Then, if the debt value is unchanged between the date t and t+ τ ($L_{t+\tau} = L_t$), as $\ln V_t \hookrightarrow \mathcal{N}\left[\ln V_0 + \left(\mu - \frac{\sigma^2}{2}\right)dt, \sigma^2 dt\right] \Leftrightarrow \ln V_{t+\tau} \hookrightarrow \mathcal{N}\left[\ln V_t + \left(\mu - \frac{\sigma^2}{2}\right)dt, \sigma^2 dt\right]$,

$$D2D = \frac{\ln\left(\frac{V_t}{L_t}\right) + \left(\mu - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} = \frac{\ln(V_t) + \left(\mu - \frac{\sigma^2}{2}\right)t - \ln(L_{t+\tau})}{\sigma\sqrt{t}} = \frac{\mathrm{E}[\ln(V_t)] - \ln(L_{t+\tau})}{\sigma\sqrt{t}}$$

Then, the D2D is negative if $\mathrm{E}[\ln(V_t)] < \ln(L_{t+\tau})$.

Their study enables to conclude that the D2D is generally a reliable measure in predicting banks failures. When it is not, the explanation can be found in the lack of transparency in financial statements and disclosed information.

Anginer, Demirguc-Kunt, Huizinga and Ma (2018) use the D2D as a measure a bank standalone risk. As Harada, Ito and Takahashi, they assume τ is equal to a year and use the past returns of the stocks. They also include the dividend rate *d*, turning the customary Black & Scholes (1973) formula applied to equity valuation into:

$$E = V.e^{-d\tau}.\Phi\left[\frac{\ln\left(\frac{V}{D}\right)+\left(r-d+\frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right] - De^{-r\tau}\Phi\left[\frac{\ln\left(\frac{V}{D}\right)+\left(r-d-\frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}\right]$$

Then, the D2D formula becomes

$$D2D = \frac{\ln\left(\frac{V}{D}\right)+\left(\mu-d-\frac{\sigma_V{}^2}{2}\right)\tau}{\sigma_V\sqrt{\tau}}$$

The *D2D* formulas show that a higher leverage and (D/V) and a higher assets' volatility reduce the *D2D* and therefore increase the risk of bankruptcy. Then these 2 variables can also be looked upon as risks' measures.

3. MES

The Marginal Expected Shortfall (MES) is presented by Acharya, Pedersen, Philippon and Richardson (2017) and by Brownlees and Engle (2017). It measures a firm's exposure to aggregate tail shocks. It has therefore a significant explanatory power for which a firm contributes to a potential crisis.

Its formula is the following one:

$$MES(C) = E_{t-1}\left(r_{it}/\sum_{j=1}^{N} w_{jt}r_{jt} < C\right)$$

where *C* is the threshold level defining a systemic event, r_{it} is the performance of institution *i*, w_{it} is its weight in the market index, *N* is the total number of institutions and E_{t-1} is the conditional expectation on all the information available in *t*-1.

It can simply be defined as the firm's average return during the 5% worst days for the market. Then, it is the BHR of the firm when the MSCI posts returns corresponding to their 5% lowest centile. Then, as underlined by Anginer, Demirguc-Kunt, Huizinga and Ma (2018), a lower MES in dicates lower returns during mar ket distress and therefore higher systemic risk.

Acharya, Pedersen, Philippon and Richardson (2010) underline that MES and leverage predict each firm's contribution to a crisis, whereas standard measures of firm-level risk, such as VaR, expected loss, or volatility have almost no explanatory power, and the beta, based on co-variance has only a modest explanatory power.

4. SRISK

SRISK is a measure of the capital to be raised by a financial institution to go on functioning normally, assuming the occurrence of a financial crisis. It has been developed by Acharya, Viral V., Brownlees, Farazmand, and Richardson (2011), by Acharya, Viral, Engle, and Richardson (2012), by Brownlees and Engle (2016) and by Engle and Ruan (2018).

SRISK is based on the *LRMES* which is defined as the Long Run Marginal Expected Shortfall. *LRMES* corresponds to the expected equity loss conditional on the market decline below the threshold *C*. Its formula is:

$$LRMES(C) = -E_t\left(r_{i,t+1:t+h}/r_{M,,t+1:h+1} < C\right)$$

where $r_{i,t+1:t+h}$ is the multi–period arithmetic firm equity return between the period *t* + 1 and the period *t* + *h*.

Acharya, Viral, Engle, and Richardson propose a proxy for LRMES:

$LRMES_{i,t}$ = 1-e$^{18.MES_{i,t}}$

The capital shortfall of firm *i* on day *t* is:

$$CS_{i,t} = k.A_{i,t} - W_{i,t} = k(W_{i,t} + D_{i,t}) - W_{i,t} = kD_{i,t} - (1-k)W_{i,t}$$

where $W_{i,t}$ is the market value of equity, $D_{i,t}$ is the book value of debt, $A_{i,t}$ is the value of quasi assets and *k* is the prudential capital ratio. Then, when the capital shortfall is negative, i.e., the firm has a capital surplus, the firm functions properly. Correlatively, when the capital shortfall is positive the firm experiences distress.

A systemic event is defined as a market decline below a threshold *C*, over a time horizon *h*, as the capital shortfall of a firm generates negative externalities if it occurs when the system is already in distress. Also, to produce a meaningful stressed capital shortfall measure, Brownlees and Engle (2016) implicitly assume that the systemic event corresponds to a sufficiently extreme scenario. In that context, SRISK is defined as the expected capital shortfall conditional on a systemic event:

$$SRISK_{i,t} = E_t\left(CS_{i,t+h}/r_{M,,t+1:h+1} < C\right)$$

$$SRISK_{i,t} = \text{k}E_t\left(D_{i,t+h}/r_{M,,t+1:h+1} < C\right) - (1-k)E_t\left(W_{i,t+h}/r_{M,t+1:h+1} < C\right)$$

Assuming that, in the case of a systemic event, debt cannot be renegotiated:

$$E_t\left(D_{i,t+h}/r_{M,,t+1:h+1} < C\right) = D_{i,t}$$

Then:

$$SRISK_{i,t} = kD_{i,t} - (1-k)(1 - LRMES_{i,t})W_{i,t}$$

Factoring by $W_{i,t}$:

$$SRISK_{i,t} = W_{i,t}\left[k\frac{D_{i,t}}{W_{i,t}} + 1 - (1-k)(1 - LRMES_{i,t}) - 1\right]$$
$$SRISK_{i,t} = W_{i,t}\left[k\frac{W_{i,t} + D_{i,t}}{W_{i,t}} - (1-k)(1 - LRMES_{i,t}) - 1\right]$$

$$SRISK_{i,t} = W_{i,t}\left[kLVG_{i,t} - (1-k)(1 - LRMES_{i,t}) - 1\right]$$

where $LVG_{i,t}$ is the leverage equal to $\frac{W_{i,t}+D_{i,t}}{W_{i,t}}$ and $LRMES = -E_t\left(r_{i,t+1:t+h}/r_{M,,t+1:h+1} < C\right)$

Merging market and balance sheet information, this formula shows that *SRISK* is a function of the size of the firm, its degree of leverage, and its expected equity devaluation conditional on a market decline.

Firms with the highest *SRISK* are the largest contributors to the undercapitalization of the financial system in times of distress. The sum of SRISK across all firms is used as a measure of overall systemic risk in the entire financial system. It can be thought of as the total amount of capital that the government would have to provide to bail out the financial system in case of a crisis.

The total amount of systemic risk in the financial system, made of *N* financial institutions, is measured as

$$SRISK_t = \sum_{i=1}^{N} max(0, SRISK_{i,t})$$

It is the total amount of capital that the government would have to provide to bail out the financial system conditional on the systemic event. Indeed, as it is difficult to raise capital in a financial crisis, the capital shortfall will either be met mostly by the taxpayer's money, or the firm will cease to function normally and may fail. For this reason, the measure is looked upon as an indicator of systemic risk in much the same way as are supervisory stress tests.

Moreover, this formula ignores the contribution of negative capital shortfalls (that is capital surpluses) as, in a crisis, it is unlikely that surplus capital will be easily mobilized through mergers or loans.

From an econometric point of view, Brownlees and R. Engle (2012) develop a multivariate approach, TARCH-DCC, to model the correlations between the returns of financial institutions. This model is used to compute the *LRMES* factor. It combines various advanced features: a common factor affects all individual returns in a time-varying framework, returns exhibit volatility clusters where periods of high volatility alternate with periods of low volatility and volatility reacts differently to an increase or a decrease based of market returns:

$$\begin{cases} r_{M,t} = \sigma_{M,t}\varepsilon_{M,t} \\ r_{i,t} = \sigma_{i,t}\rho_{i,t}\varepsilon_{M,t} + \sigma_{i,t}\sqrt{1-\rho_{i,t}}\xi_{i,t} \\ (\varepsilon_{M,t}, \xi_{i,t}) \hookrightarrow \mathcal{D} \end{cases}$$

where $\varepsilon_{M,t}$ and $\xi_{i,t}$ represent shocks that are independent and identically distributed over time with zero mean, unit variance and zero covariance. Moreover, $\mathcal{D}$ is the (unspecified) cumulative distribution function of innovation. The first formula evidences that the market return, $r_{M,t}$, is decomposed into a volatility factor, $\sigma_{M,t}$, and an innovation factor $\varepsilon_{M,t}$. The second formula shows that the return of institution *i*, $r_{i,t}$, is a mixture of a common term, $\varepsilon_{M,t}$, and a specific term, $\xi_{i,t}$. The two terms are related to each other through a dynamic correlation, $\rho_{i,t}$, and a dynamic volatility, $\sigma_{i,t}$, that both depend on institution *i*. $\xi_{i,t}$ represents the innovation specific to institution *i*. The volatility terms, $\sigma_{M,t}$, and $\sigma_{i,t}$ are modeled according to a TGARCH specification. TGARCH model leads to get financial time series with volatility clustering and threshold effects. The specification is:

$$\begin{cases} \sigma_{M,t}^2 = \omega_{MG} + \alpha_{MG} r_{M,t-1}^2 + \gamma_{MG} r_{M,t-1}^2 1_{r_{M,t-1<0}} + \beta_{MG}\sigma_{M,t-1}^2 \\ \sigma_{i,,t}^2 = \omega_{iG} + \alpha_{iG} r_{i,t-1}^2 + \gamma_{iG} r_{i,t-1}^2 1_{r_{i,t-1<0}} + \beta_{iG}\sigma_{i,t-1}^2 \end{cases}$$

This shows that the volatility of returns has a time dependency β and increases with the squared value of the return. Moreover, the response is asymmetric because the volatility does not respond identically in the case of a loss or a gain since the threshold effects depend on the sign of the returns.

Brownlees and R. Engle (2012) write down:

$$\begin{pmatrix} r_{i,t} \\ r_{M,t} \end{pmatrix} \Bigg| \mathcal{F}_{\text{t-1}} \hookrightarrow \mathcal{D}\left(0, \begin{bmatrix} \sigma_{i,,t}^2 & \rho_{i,t}\sigma_{i,t}\sigma_{M,t} \\ \rho_{i,t}\sigma_{i,t}\sigma_{M,t} & \sigma_{M,t}^2 \end{bmatrix}\right)$$

where $\mathcal{F}_{\text{t-1}}$ is the available information at time *t*-1.

The values of SRISK for historical periods that can be chosen are provided on the V-LAB website of New York Stern University.

5. Customary control variables

a. ROA

The return on assets (ROA) is often calculated, for financial institutions as the net income divided by the total assets. Then, a higher ROA is consistent with a lower solvency risk

b. P/BV

The price (or market) to book value is market cap divided by the equity group share. Then a higher P/BV is consistent with a lower solvency risk as the market cap takes the solvency equity into account, ie core equity tier 1 for banks and own funds for insurance companies

c. Size

Research paper are used to basing the size on the natural log of total assets. Its impact on risk is ambivalent. Indeed, as underlined by Anginer, Demirguc-Kunt, Huizinga and Ma (2018), larger banks could pursue riskier strategies if they are deemed to be too big to fail, but they could also be less risky thanks to a better diversification

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